MATH-2400 NAME: _____

Instructor: Joe Klobusicky Thursday, October 4, 2018

Exam 1

Please answer all questions, showing your work in detail and giving reasons for your conclusions.

You may use **both sides of a** (two-sided) $8\frac{1}{2} \times 11$ sheet
in your own handwriting, but no other notes, books, computers, calculators, cell phones, or other references or communication tools are permitted.

Please circle your section: **13** **14** **15** **16**

Problem	Points
1/16 pts.	
2/30 pts.	
3/30 pts.	
4/24 pts.	
TOTAL	

1. (a) [16 pts.] Find the solution of the initial-value problem

$$ty(t)' - 2y(t) - t = 0, \qquad y(1) = 1.$$

Sketch a solution for the equation.

2. The Troy Power Company has decided to switch to nuclear power to run its facility. Specifically, it relies on a supply of the radioactive (and fictional) material of Albanium, with a decay rate of 3/year. Moreover, the manager of the company has decided to continuously add Albanium at a constant rate of 3kg/year. Suppose the company at time $t = 0$ begins with 2 kg of Albanium.

(a) [6 pts.] Write down an initial value problem (differential equation + initial conditions) for the amount $Q(t)$ of Albanium at time $t \geq 0$.

(b) [12 pts.] Solve the initial value problem.

(c) [6 pts.] If the amount of Albanium reaches 5 kg, the facility goes into panic mode. When, if ever, does panic mode happen?

(d) [6 pts.] The manager chose to add 3kg of Albanium a year because he thought it would balance out the loss due to the decay rate of 3/year. This is incorrect reasoning, however. What value should the manager choose in order for the amount of Albanium to remain constant at 2 kg for all times?

3. Consider an autonomous first order differential equation, given by

$$\frac{dM}{dt} = M^2(M-2)^2.$$

(a) [12 pts.] Draw the phase line and plot the M' vs. M graph. What is the value and stability type for each of the equilibria?

(b) [12 pts.] Sketch representative integral curves on the t, M plane.

(c) [6 pts.] What happens to $M(t)$ as $t \to \infty$ if

(i) $M(0) = -2$,

(ii) $M(0) = 1$,

(iii) $M(0) = 2$

4. (a) [12 pts.] Compute the solution of the initial value problem

$$4y'' - 4y' + y = 0 \qquad y(0) = 0, \quad y'(0) = 1.$$

4. (b) [12 pts.] Compute the solution of the initial value problem

$$y'' - 4y' + 20y = 0, \qquad y(0) = 0, \quad y'(0) = -1.$$