MATH 2400: Suggested Homework 4

12. A mass weighing 16 lb. stretches a spring 3 in. The mass is attached to a viscous damper with a damping constant of 2 lb-sec/ft. If the mass is set in motion from its equilibrium position with a downward velocity of 3 in/sec, find its position u at any time t. Draw u versus t. Determine when the mass first returns to its equilibrium position. Also, find the time τ such that $|u(t)| < 0.01$ in for all $t > \tau$. (Recall that g = 32 ft/sec2 .)

The positions of certain mass-spring systems satisfy the following initial value problems:

$$13. \quad \ddot{u} + 2u = 0, \qquad u(0) = 0, \qquad \dot{u}(0) = 2.$$

$$14. \quad \ddot{u} + \frac{1}{4}\dot{u} + 2u = 0, \qquad u(0) = 0, \qquad \dot{u}(0) = 2.$$

(a) Find the solutions of these initial value problems.

(b) Draw u versus t on the same axes.

(c) Draw $\dot{u}$ versus u; that is, draw $u(t)$ and $\dot{u}(t)$ parametrically with t as the parameter. What is the direction on this curve as t increases? Identify several corresponding points on the curves in parts (b) and (c).

15. A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external force of $10\sin(t/2)$ N (newtons=kg $\cdot$ m / sec ^{2}) and moves in a medium that imparts a viscous force of 2 N when the speed is of the mass is 4 cm/sec.

(a) If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/sec, formulate the initial value problem describing the motion of the mass, and find its solution.

(b) Identify the transient and steady-state parts of the solution.

(c) Draw the graph of the solution, as well as the steady-state solution.

(d) If the given external source is replaced by a force $2\cos\omega t$ of frequency ω, find the value of ω for which the amplitude of the forced response is maximum.

16. Consider a vibrating system described by the initial value problem:

$$\ddot{u} + \frac{1}{4}\dot{u} + 2u = 2\cos\omega t, \qquad u(0) = 0, \qquad \dot{u}(0) = 2.$$

(a) Determine the steady part of the solution of this problem.

(b) Find the amplitude R of the steady state solution in terms of ω.

(c) Find the maximum value of R and the frequency ω for which it occurs.

(d) Draw R versus ω.

Find the general solutions of the following Euler's equations:

17. $x^2y'' + 4xy' + 2y = 0$

18. $(x-1)^2y'' + 8(x-1)y' + 12y = 0.$

19. $2x^2y'' - 4xy' + 6y = 0$

20. $x^2y'' - 5xy' + 9y = 0.$

21. Find the solution of the initial-value problem

$$4x^2y'' + 8xy' + 17y = 0, \qquad y(1) = 2, \quad y'(1) = -3.$$

Sketch this solution, and discuss its behavior as $x \to 0$.

In each of the following problems use variation of parameters to find the general solution of the given differential equation:

22. $y'' + y = \tan x$

23. $y'' + 9y = 9\sec^2 3x$

24. $y'' + 4y = 3\csc 2x.$