MATH-2400 NAME: _____

Instructor: Joe Klobusicky Tuesday, February 27, 2018

Exam 1

Please answer all questions, showing your work in detail and giving reasons for your conclusions.

You may use **one side of a** (two-sided) $8\frac{1}{2} \times 11$ sheet in your own handwriting, but no other notes, books, computers, calculators, cell phones, or other references or communication tools are permitted.

Please circle your section: **17** **18** **19** **20**

Problem	Points
1/24 pts.	
2/26 pts.	
3/26 pts.	
4/24 pts.	
TOTAL	

1. (a) [12 pts.] Find the solution of the initial-value problem

$$y' + \cos(x)y = \cos(x), \qquad y(0) = 0.$$

1. (b) [12 pts.] Find the general solution of

$$y'' + 2y' + 3y = 0.$$

2. Sally Saver opens a checking account at time $t = 0$, with an initial amount of 1 dollar. Suppose the checking account grows with a continuously compounded interest of $r = .1$ per year. Further assume that money is continuously withdrawn from the account at .5 dollars per year.

(a) [12 pts.] Write down an initial value problem describing the total value $S(t)$ of Sally's checking account after t years.

(b) [10 pts.] Solve this initial value problem.

(c) [4 pts.] When, if ever, does Sally go broke (meaning that she has zero dollars in her account)?

3. Consider an autonomous first order differential equation, given by

$$\frac{dM}{dt} = (M-5)^2(M+1).$$

(a) [12 pts.] Draw the phase line and plot the M' vs. M graph. What is the value and stability type for each of the equilibria?

(b) [10 pts.] Sketch representative integral curves on the t, M plane.

(c) [4 pts.] Given an initial value of $M(0) = 0$, what happens to $M(t)$ as $t \to \infty$?

4. (a) [12 pts.] Determine the proper **form** for the general solution $y(x)$ of the following second order equation (you do not need to evaluate any undetermined constants):

$$y'' + y = x(1 + \sin(x))$$

4. (a) [12 pts.] Compute the general solution of

$$y'' + y' = x^2.$$

(Here, you need to evaluate undetermined coefficients for the particular solution)