MATH-2400 Sections 17–18 NAME: _____

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Exam #3

Please show all work. Each question is worth 15 points.
Do not use text books, notes, calculators, or other aids.
You may use both sides of one $8\frac{1}{2} \times 11$" crib sheet.

PROBLEM #	POINTS
1	
2	
3	
4	
Total	

1. [15 pts.] Given that the eigenvalues and eigenfunctions for the problem

$$X'' + \sigma X = 0, \qquad X'(0) = 0, \quad X'(\pi) = 0$$

are

$$\sigma_n = n^2, \qquad X_n(x) = \cos(nx), \qquad n = 0, 1, , \ldots$$

use separation of variables to find the functions of $u_n(x,t)$ that satisfy the heat equation

$$u_t = u_{xx}, \qquad 0 < x < \pi, \quad t > 0,$$

and the boundary conditions

$$u_x(0,t) = 0, \quad u_x(\pi,t) = 0, \qquad t > 0.$$

Then, use those functions to find the full solution $u(x,t)$ to the heat equation subject to the initial condition $u(x,0) = 3 - \cos(x) + 10\cos(7x)$.

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2. [15 pts.] Consider the initial/boundary value PDE

$$z_{xx} + \cos(x)z = z_t, \qquad 0 < x < l, \qquad t > 0,$$

$$z_x(0,t) = 0, \quad z(l,t) = 0, \qquad t > 0.$$

$$z(x,0) = x^3, \qquad 0 < x < l.$$

(a) [10 pts.] Assuming a solution of the form $z(x,t) = X(x)T(t)$, find ordinary differential equations satisfied by $X(x)$ and $T(t)$.

(b) [5 pts.] One of the two equations you derived in part (a) corresponds to an eigenvalue problem. Use the boundary conditions for $z(x,t)$ to derive the boundary conditions for this eigenvalue problem, and state this problem (i.e. give the boundary value problem, but you don't have to solve it).

3. [15 pts.] Find the displacement $u(x,t)$ of an elastic string of length l that is fixed at its ends and is set in motion by pulling the string at its center and releasing. In this case, $u(x,t)$ satisfies the wave equation

$$u_{tt} = a^2 u_{xx},$$

with the boundary conditions

$$u(0,t) = 0, \qquad u(l,t) = 0,$$

and the initial condition

$$u(x,0) = f(x), \qquad u_t(x,0) = 0,$$

with $f(x)$ defined by

$$f(x) = \begin{cases} x, & 0 \le x \le l/2, \\ l - x, & l/2 \le x \le l. \end{cases}$$

(Hint: Here, you may use the fact that

$$u(x,t) = \sum_{n=1}^{\infty} (c_n \sin(n\pi a t/l) + d_n \cos(n\pi a t/l)) \sin(n\pi x/l)$$

is a general solution to the wave equation under fixed endpoints.)

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4. [15 pts.] Find all the eigenvalues and eigenfunctions of the boundary value problem

$$X'' + \sigma X = 0, \qquad X'(0) = 0, \qquad X(5) = 0$$

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