MATH-2400

NAME: KEY

Instructor: Joe Klobusicky

Tuesday, February 27, 2018

Exam 1

Please answer all questions, showing your work in detail and giving reasons for your conclusions.

You may use **one side of a** (two-sided) $8\frac{1}{2} \times 11$ sheet in your own handwriting, but no other notes, books, computers, calculators, cell phones, or other references or communication tools are permitted.

Please circle your section: **17** **18** **19** **20**

Problem	Points
1/24 pts.	
2/26 pts.	
3/26 pts.	
4/24 pts.	
TOTAL	

1. (a) [12 pts.] Find the solution of the initial-value problem

$$y' + \cos(x)y = \cos(x), \qquad y(0) = 0.$$

$\mu(x) = e^{\int \cos(x)dx} = e^{\sin(x)}$

$[\mu(x)y]' = \cos(x)\mu(x)$

$e^{\sin(x)}y = \int \cos(x)e^{\sin(x)}dx = e^{\sin(x)} + C$

$y(x) = 1 + Ce^{-\sin(x)}$.

For IC, $0 = y(0) = 1 + Ce^{-\sin(0)} = 1 + C \Rightarrow C = -1$

Sol'n then $y(x) = 1 - e^{-\sin(x)}$

1. (b) [12 pts.] Find the general solution of

$$y'' + 2y' + 3y = 0.$$

Ch. Eqn r^2+2r+3 Roots $r_1, r_2 = \frac{-2 \pm \sqrt{4-12}}{2} = -1 \pm \sqrt{2}i$

Gen. Sol'n

$y(x) = C_1 e^{-x} \sin(\sqrt{2}x) + C_2 e^{-x} \cos(\sqrt{2}x)$

2. Sally Saver opens a checking account at time $t = 0$, with an initial amount of 1 dollar. Suppose the checking account grows with a continuously compounded interest of $r = .1$ per year. Further assume that money is continuously withdrawn from the account at .5 dollars per year.

(a) [12 pts.] Write down an initial value problem describing the total value $S(t)$ of Sally's checking account after t years.

(b) [10 pts.] Solve this initial value problem.

(c) [4 pts.] When, if ever, does Sally go broke (meaning that she has zero dollars in her account)?

a) $S'(t) = .1S - .5$

b) $S' - .1S = -.5 \qquad \mu(t) = e^{-\frac{t}{10}}$

$[\mu(t)S]' = -\frac{1}{2}e^{-\frac{t}{10}}$

$e^{-\frac{t}{10}}S = 5e^{-\frac{t}{10}} + C \Rightarrow S(t) = 5 + Ce^{\frac{t}{10}}$

$S(0) = 1 \Rightarrow \qquad 1 = S(0) = 5 + C \Rightarrow C = -4$, so $S(t) = 5 - 4e^{\frac{t}{10}}$

c) Broke when $\quad 0 = S(t) = 5 - 4e^{\frac{t}{10}}$

$e^{\frac{t}{10}} = \frac{5}{4} \Rightarrow t = 10\log\left(\frac{5}{4}\right)$

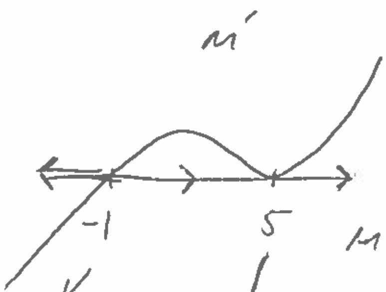
3. Consider an autonomous first order differential equation, given by

$$\frac{dM}{dt} = (M-5)^2(M+1).$$

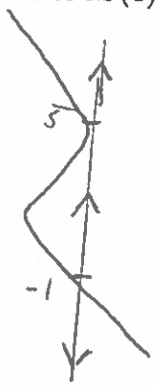
(a) [12 pts.] Draw the phase line and plot the M' vs. M graph. What is the value and stability type for each of the equilibria?

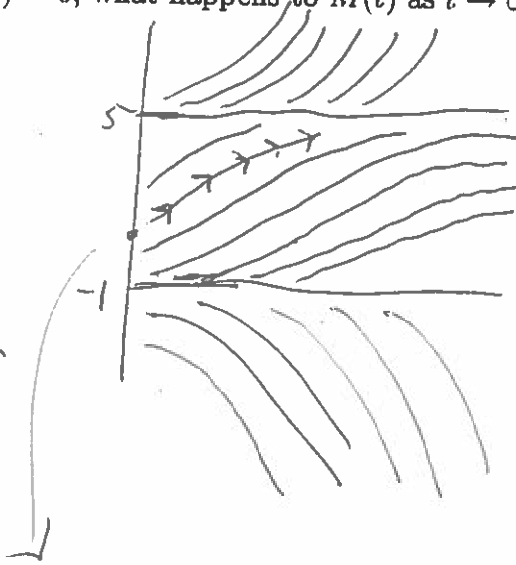
(b) [10 pts.] Sketch representative integral curves on the t, M plane.

(c) [4 pts.] Given an initial value of $M(0) = 0$, what happens to $M(t)$ as $t \to \infty$?



unstable semi-stable





At $M(0)=0$ solution approaches

5 as $t \to \infty$

4. (a) [12 pts.] Determine the proper **form** for the general solution $y(x)$ of the following second order equation (you do not need to evaluate any undetermined constants):

$$y'' + y = x(1 + \sin(x)) = x + x\sin(x)$$

Comp. Sol $y_c'' + y_c = 0$ $r^2+1 = 0 \Rightarrow r = \pm i$

$y_c(x) = C_1 \sin(x) + C_2 \cos(x)$

For x: $y_{P_1} = Ax + B$

$\alpha + i\beta = 0$

For $x\sin(x)$: $y_{P_2} = x[(Dx+E)\sin(x) + (Fx+G)\cos(x)]$

$\alpha + i\beta = i$, so $s=1$

Then Form is

$y(x) = y_c + y_{P_1} + y_{P_2} = C_1\sin(x) + C_2\cos(x)$
$+ Ax + B + x[(Dx+E)\sin(x) + (Fx+G)\cos(x)]$.

4. (a) [12 pts.] Compute the general solution of

$$y'' + y' = x^2.$$

(Here, you need to evaluate undetermined coefficients for the particular solution)

Comp $y_c'' + y_c' = 0$ Ch. Eqn $r^2 + r = 0$
$\Rightarrow r(r+1) = 0 \quad r = 0, -1$

Comp Sol $y_c = C_1 x^0 + C_2 x^{-1} = C_1 + \frac{C_2}{x}$

Part $\alpha + i\beta = 0$ then $y_p = x[Ax^2 + Bx + C] = Ax^3 + Bx^2 + Cx$
So $\underline{s = 1}$

Match coeffs $y_p' = 3Ax^2 + 2Bx + C \quad y_p'' = 6Ax + 2B$

$6Ax + 2B + 3Ax^2 + 2Bx + C = x^2$

$x^2: \quad 3A = 1 \qquad A = \frac{1}{3}$

$x: \quad 6A + 2B = 0 \quad \Rightarrow \quad 6(\frac{1}{3}) + 2B = 0 \Rightarrow B = -1$

$1: \quad 2B + C = 0 \qquad C = 2.$

So $y_p = \frac{1}{3}x^3 - x^2 + 2x$

$y = y_p + y_c.$