

MATH-2400 (Sections 17-18, Spring 2017) NAME: _____

Instructor: Joe Klobusicky SECTION: _____

Practice Exam

- Please show all work.
- Do not use text books, notes, calculators, cell phones, or other aids.
- You may use one side of an $8\frac{1}{2} \times 11$ " crib sheet.
- Each problem is 10 out of a total of 60 points.

PROBLEM #	POINTS
1	
2	
3	
4	
5	
6	
Total	

1. Solve the initial-value problem

$$(x + 1)^2 y' - (x + 1)y = -2, \quad y(0) = 0,$$

for y as a function of t and sketch the solution.

2. Suppose that 100 mg of thorium-234 are initially present in a closed container, and that thorium-234 is added to the container at a constant rate of 1 mg/day.

- (a) Find the amount $Q(t)$ of thorium-234 in the container at any time, given that its decay rate is 0.01 days^{-1} (Hint: the change in $Q(t)$ is the difference between rate added and rate decayed).
- (b) Find the limiting amount Q_1 of thorium-234 in the container as $t \rightarrow \infty$.
- (c) If thorium-234 is added to the container at a rate of k mg/day, find the value of k that is required to maintain a constant level of 100 mg of thorium-234.

3. Find the general form for the solution y of the initial-value problem

$$y'' + y' + 2y = \sin(3x)e^x(1 + x) + x.$$

Note: By general form, this means you may leave unsolved constants in your solution.

4. Consider the problem

$$\frac{dN}{dt} = N^2 - 2N.$$

Sketch dN/dt versus N , determine all possible fixed equilibria and their stability type (stable, unstable, semi-stable), and draw some representative integral (solution) curves in the $t - N$ plane.

5. Compute the solution of the initial-value problem

$$x^2y'' + xy' + 4y = 0, \quad x > 0.$$

As $x \rightarrow \infty$, will the solution $y(x)$ approach ∞ , $-\infty$, or neither? Explain why? (Note: this answer should not depend on initial conditions!)

6. Compute the solution of the initial-value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$

and sketch this solution.

MATH-2400 Test #1 NAME: _____

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(Extra page for work)