

MATH-2400 Sections 17–18

NAME: \_\_\_\_\_

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## Sample Exam #2

Please show all work. Each question is worth 10 points.

Do not use text books, notes, calculators, or other aids.

You may use one  $8\frac{1}{2} \times 11$ " crib sheet.

PROBLEM #	POINTS
1	
2	
3	
4	
5	
Total	

1. Using variation of parameters, find the general solution of the equation

$$x^2y'' - 2xy' + 2y = x^2, \quad x > 0$$

2. Some mass-spring system is described by the equation

$$\ddot{y} + ky = \cos t.$$

- (a) Find the solution of this problem for  $k < 0$  and  $k = 0$ .
- (b) When  $k > 0$ , for which value(s) of  $k$  should we expect  $y(t)$  to grow indefinitely in time as  $t \rightarrow \infty$ ? You can either give a physical explanation, or explain what the solution looks like for special values of  $k$ .

3. Find the general solution of the linear system

$$\dot{\mathbf{x}} = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \mathbf{x},$$

and sketch the phase portrait in the  $x_1 - x_2$  plane. Along which vector(s) do solutions tend towards the origin as  $t \rightarrow \infty$ ?

4. Compute the general solution of the system

$$\dot{\mathbf{x}} = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{x},$$

and sketch the phase portrait in the  $x_1 - x_2$  plane. What happens to the solutions as  $t \rightarrow \infty$ ?

5. For any real  $a_1 > a_2 > 0$ , compute the eigenvalues and eigenvectors of the *diagonal* matrix

$$A = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}$$

**Bonus:** What about the *antidiagonal* matrix

$$B = \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}.$$

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